Underwater Application of Spectral Analysis of Surface Waves

L. Paoletti[1], E. Mouton[2], and M. Maraschini[3]

[1]D'Appolonia S.p.A, Via S. Nazaro 19, 16145 Genova, Italy, email lorenzo.paoletti@dappolonia.it
[2]SismOcean S.A.S, rue de la Cité Blanche 23, 34200 Sete, France, email edouard.mouton@sismocean.com
[3]D'Appolonia, S.p.A, Via S. Nazaro 19, 16145 Genova, Italy, email margherita.maraschini@dappolonia.it

Abstract

The spectral analysis of surface waves (SASW) technique is frequently used on land to measure the shear wave velocity profile. This paper presents an application of the methodology for underwater site investigations. Field data are collected using a bottom-towed acoustic source and a streamer of hydrophones. Multichannel data are processed by slant-stacking and fast Fourier transform to find the relation between phase velocity and frequency. Inversion processing of these dispersion curves is used to generate a continuous profile of soil stiffness below sea floor. An example case of a site investigation in the Adriatic Sea is presented. Comparison of shear wave velocity profiles obtained with downhole seismic cone and the underwater surface waves technique shows good agreement confirming the reliability of the multichannel analysis of surface waves (MASW) underwater.

Introduction

Properties of Surface Waves

In an infinite homogeneous continuum, only compression and shear waves are possible. The presence of a boundary in the continuum allows for a third type of wave. If the interface is between air and solid (i.e. a free boundary) the surface wave is of the Rayleigh type. If the interface is between water and solid the surface wave is of the Scholte type. The particle motion of a surface wave is both compressional and rotational. Surface waves have a number of advantages as a geophysical survey technique as summarized in Table 1 (Richart et al., 1970):

Table 1. Properties and Advantages of Surface Waves

Property	Advantages
For a vertically oscillating footing on the surface of a halfspace, about 2/3 of the energy is distributed to the surface waves, and the remaining 1/3 is divided between compression and shear waves.	Most of the source energy is in the surface waves.
The geometrical damping at the surface is proportional to the square root of distance for surface waves, and to the distance squared for the compression and shear waves.	Surface waves are considerably less dampened than other waves at shallow depths.
Surface waves are slower than compression and shear waves.	Signal corresponding to surface waves is easily identified.
Penetration of surface waves into the medium is about one wavelength.	Long wavelengths (low frequencies) travel deeper than short wavelengths (high frequencies).

The amplitude of vertical displacement as a function of wavelength is qualitatively shown in Figure 1. In the case of a layered medium, high frequencies (i.e. short wavelength) will travel with velocity of the shallow layers, while low frequencies (i.e. long wavelength) will travel with velocities corresponding to the deeper layers. Consequently, the frequency content of the surface wave will change with distance from the source. This property of the surface waves is called "dispersion". The spectral analysis of surface waves (SASW) utilizes the property of dispersion to obtain the shear wave velocity profile below ground surface or seabed.

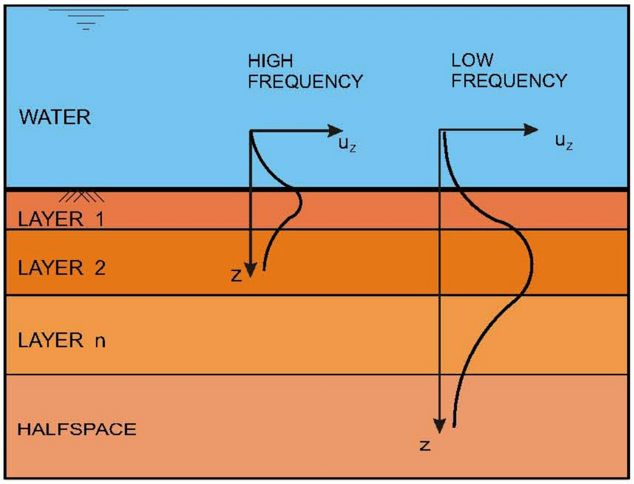

Figure 1. Amplitude of vertical displacement and penetration of surface waves at different frequencies

Surface Wave Equations

The motion of surface wave can be described solving a system of equations representing the indefinite equilibrium of the medium, its constitutive laws, the boundary conditions modeling the water layer and the halfspace, and the continuity of stress and strain tensors at the interfaces (see for example Lay and Rix, 1998; Foti, 2000; Strobbia, 2003). A horizontal layered geometry is considered.

The indefinite equilibrium equation is:

$$\sigma_{ij,j} + \rho f_i = \rho \ddot{u}_i$$

where σ_{ij} is the stress tensor, ρ is the material density, u is the displacement and f_i is the body force per unit mass.

The soil is modeled as linear elastic material, which is a reasonable assumption considering the small strains generated by the surface wave. For the soil, stresses are related to strain by means of the following equation:

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij}$$

where λ and μ are Lamé's parameters, ε_{ij} is the strain tensor and δ_{ij} is the Kronecker delta.

The water layer is modeled as inviscid and compressible, i.e. (Lee, 1996):

$$p = k\varepsilon_{kk}$$

where k is the bulk modulus of the water and $\sigma_{ij} = -p\delta_{ij}$.

The Scholte wave equation can be found resolving an implicit eigenvalue problem assuming an exponential form of the solution. The equation describing the Scholte waves, in terms of the wave number *k*, is (Guilbot, 1994):

$$4k^2 q s - (k^2 + s^2)^2 = \frac{\rho_w}{\rho} s^4 \frac{q}{q_w} \tanh(q_w h_w)$$

where ω is the angular velocity, $k = \omega / v_{sw}$, $q = \omega / v_p$, $s = \omega / v_s$, $q_w = \omega / v_{p\,water}$ and h_w is the height of the water column. Symbols v_{sw}, v_p, v_s, and $v_{p\ water}$ are velocity of the surface wave, compression and shear wave velocity in soil, and compression wave velocity in water, respectively. The right term of the equation is zero for the case of surface waves onshore.

Another important feature of surface waves propagation in layered media is the existence of several modes of propagation (Achenbach 1984).

Inversion Processing of Surface Wave Data

Processing of seismic surface waves data consists of the following steps (Figure 2):

- Data editing for spikes;
- Transformation of data from *x-t* plane to *v-f* plane where *x* is distance, *t* is time, *v* is surface wave velocity, and *f* frequency by means of slant staking and FFT;
- Picking the dispersion curve *v=v(f)*;
- Inversion from *v=v(f)* to *v=v(z)* where z is the depth below seafloor (Figure 2). The inversion process is repeated until the model and picked dispersion curve show a good match. The water layer is modeled considering the speed of sound, the density of water, and water depth. The soil is modeled as a series of horizontal layers and a substrate of infinite thickness. The layers are defined by the thickness, the density, and the P and S wave velocities.

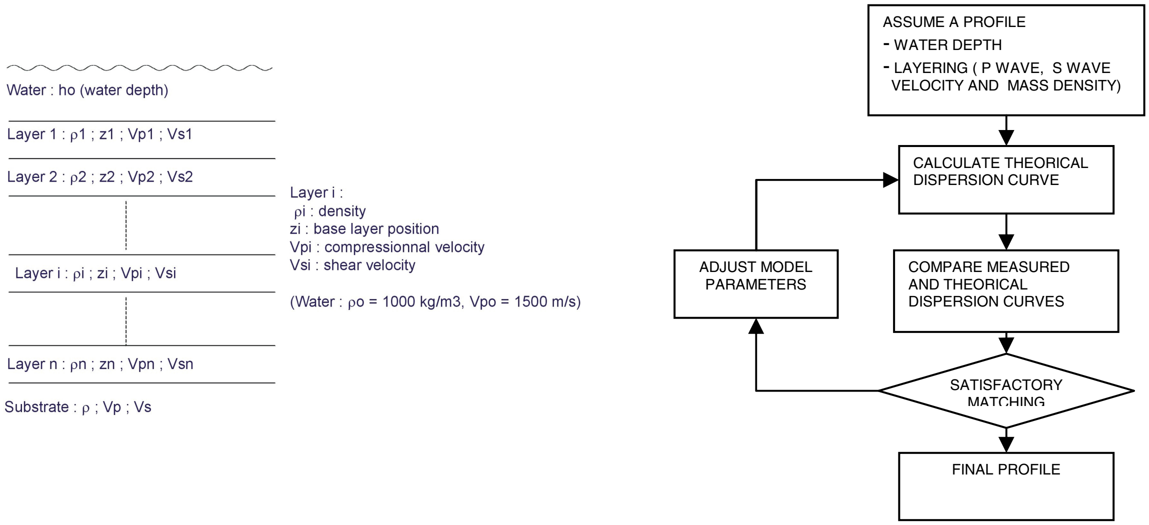

Figure 2. Inversion process scheme

Example Case Study

This chapter describes an application of the underwater multichannel analysis of surface waves technique in the Adriatic Sea. The shear wave velocity profile was needed for the seismic design of a 2 km bridge offshore Croatia. The shear wave velocity was also measured in 5 boreholes using a downhole seismic CPT.

Equipment and Method of Operations

The equipment utilized for the underwater surface waves survey is shown in Figure 3. Survey equipment included an airgun (20 to 40 in^3) and a streamer of 24 hydrophones at 5 m spacing. The equipment was installed on a dedicated geophysical vessel. An air compressor, with capacity of 12 m^3/h at 120 bars was used to fill two air bottles as air buffer. Low frequency hydrophones were used as the site comprised a thick sequence of normally consolidated to lightly overconsolidated clays.

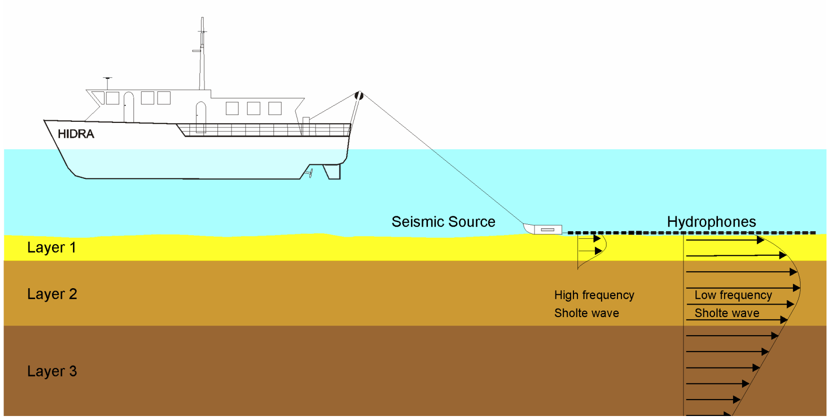

Figure 3. Survey equipment

The streamer and the source were deployed to the seabed with the vessel slowly moving along the survey alignment, and then dragged on the seabed at a speed of about 2 knots. Airgun shots were made at 20 to 50 m intervals. Data quality was checked onboard to ensure good seismic records.

Data Examples

Figure 4 shows the filtered seismic record and the dispersion pattern at a shotpoint along the bridge route. The surface wave is well represented in the central portion of the sonogram (left panel of Figure 4). The dispersion pattern shows a well defined curve (right panel of Figure 4).

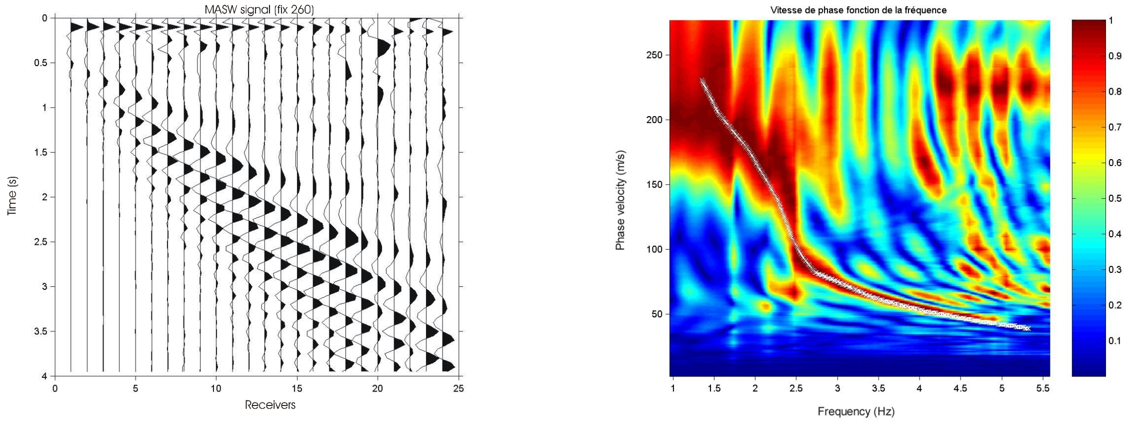

Figure 4. Seismic data and dispersion pattern

The results of data processing are shown in Figure 5: the first panel shows the picked and modeled dispersion patterns. The model dispersion pattern matches the picked dispersion curve well, particularly for the higher frequencies. The model slightly overpredicts the shear wave velocities at very low frequencies (greater depths). The right panel shows the shear wave velocity profile corresponding to the model dispersion pattern.

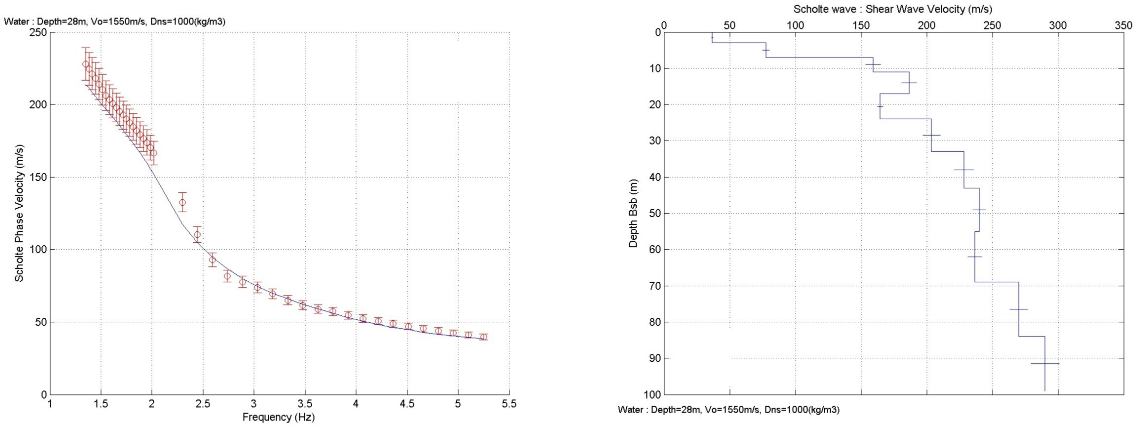

Figure 5. Dispersion curve and shear wave velocity profile

Comparison with Seismic CPT

The typical method to measure shear wave velocity underwater is the seismic cone penetration test. A cone equipped with a triaxial accelerometer is pushed into the soil to the required depth. A shear wave is generated at the seabed by striking a hammer on a reaction frame resting on the seabed, and the wave arrival is recorded. Subsequently, the cone is advanced and the test repeated. The ratio between test depth and delay gives the average shear wave velocity in the depth interval. This method is generally accepted by the industry, however it requires considerable vessel time to drill the borehole. Moreover, the seismic cone measurements are representative of the borehole location only, while in many cases a quasi-continuous survey would be preferred. The underwater surface wave survey is considerably less expensive and provides the same level of accuracy, as shown in the Figure 6 below.

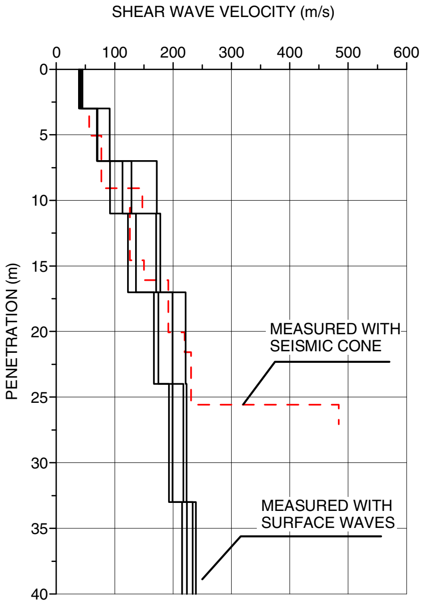

Figure 6. Comparison of seismic CPT and surface waves survey data at one borehole location

Conclusions

This paper presents an application of multichannel spectral analysis of surface waves (MASW) for underwater sites. Data were collected by means of a streamer of hydrophones from a small survey vessel. The data were processed to obtain the dispersion curve, that is the function of the phase wave velocity of the surface wave with frequency. Numerical inversion was used to derive the shear wave velocity profile with penetration below the seafloor.

The comparison between underwater MASW and seismic CPT shows good agreement in the results. The underwater surface waves survey offer several advantages with respect to the seismic cone measurements, including:

- underwater MASW survey can be conducted in a quasi-continuous mode, providing an almost continuous shear wave velocity profiling along a route. Proceeding at a survey speed of about 2 knots, several km of survey can be executed in one day of operations. Anomalous site conditions can be identified during the survey;
- there is no need to drill a borehole to advance the cone in the ground. A conventional geophysical vessel can be used. Consequently, survey costs are considerably lower.

With respect to the conventional offshore geophysical surveys, underwater MASW offer the following advantages:

- penetration of surface waves is not limited by the water depth, while conventional methods are affected by multiple reflections which mask the acoustic signal in shallow water;
- surface waves penetrate in the soil also in the cases of a stiff layer overlying soft layers, and in presence of gas charged sediments.

These advantages make the underwater SASW particularly attractive in offshore surveys.

References

Achenbach, J. D. (1984). *Wave Propagation in Elastic Solids*, North-Holland Amsterdam, Netherlands.

Lee, B. (1996). *Analytical Studies of Surface Wave Propagation along the Seafloor for Application to Spectral Analysis of Surface Waves (SASW) Testing*, PhD Dissertation, The University of Texas at Austin, May.

Foti, S. (2000). *Multistation Methods for Geotechnical Characterization using Surface Waves*, PhD Thesis, Polytechnic of Turin, February.

Guilbot, J. (1994). *Caractérisation acoustique de fonds sédimentaires marins par étude de la dispersion de célérité des ondes d'interface de type Stoneley-Scholte*. PhD Dissertation, The Science University of Lyon, France.

Lai, C. G. and Rix, G. J. (1998). *Simultaneous Inversion of Rayleigh Phase Velocity Attenuation for Near-Surface Site Characterization*, Georgia Institute of Technology, July.

Richart, F. E., Woods, R. D. and Hall J. R. (1970). *Vibration of Soils and Foundations,* Prentice-Hall.

Strobbia, C. (2003). *Surface Wave Methods - Acquisition, Processing and Inversion*, Extract from PhD Thesis, Polytechnic of Turin.